An Efficient Algorithm for Learning Event-Recording Automata

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October 14, 2011

Motivation

Why is learning of models important?

- Automatic inference or construction of abstract models

Outline

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The L* Algorithm

Timed Language and Event-Recording Automata

The TL* Algorithm

Conclusion and Future Work

The L* Algorithm

The L* algorithm is a formal method to learn a minimal DFA that accepts an unknown language U over an alphabet Σ.

- D. Angluin, "Learning regular sets from queries and counterexamples," *Information and Computation* 75 (1987), no. 2, 87-106.

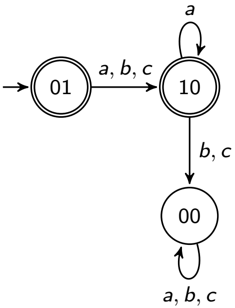
The L$*$ algorithm interacts with a Minimal Adequate *Teacher*

- *membership query*
  - Is a string in the unknown language U?
- *candidate query*
  - Does a DFA accept the unknown language U?

The L* Algorithm (cont.)

The unknown language $U = (a \mid b \mid c) \cdot a^*$

	λ	c
λ	0	1
a	1	0
b	1	0
c	1	0
$a \cdot a$	1	0
$a \cdot b$	0	0
$a \cdot c$	0	0
$ab \cdot a$	0	0
$ab \cdot b$	0	0
$ab \cdot c$	0	0



Outline

Timed Language

Let Σ be a finite alphabet.

For every symbol (event) $a \in \Sigma$, we use x_a to denote the *event-recording clock* of the symbol a

- x_a records the time elapsed since the last occurrence of the symbol a
- We use C_Σ to denote the set of event-recording clocks over Σ

An *atomic clock guard* τ is an inequation of the form $x_a \sim n$ for $x_a \in C_\Sigma$, $\sim \in \{<, \leq, >, \geq\}$, and $n \in N$.

- A *clock guard* g is a conjunction of atomic clock guards.
- We use G_Σ to denote the set of clock guards over C_Σ

A *guarded word* is a sequence $w_g = (a_1, g_1)(a_2, g_2) \cdots (a_n, g_n)$ where $a_i \in \Sigma$ for $i \in \{1, 2, \ldots, n\}$ and $g_i \in G_\Sigma$ is a clock guard.

Event-Recording Automata

An *event-recording automaton* (ERA) $D = (\Sigma, L, l_0, \delta, L^f)$ consists of

- a finite input *alphabet* Σ
- a finite set of *locations* L
- an *initial location* $l_0 \in L$
- a *transition function* $\delta : L \times \Sigma \times G_\Sigma \mapsto 2^L$
- a set of *accepting locations* $L^f \subseteq L$

Each event-recording clock $x_a \in C_\Sigma$ is implicitly and automatically *reset* when a transition with event a is taken.

Event-Recording Automata (cont.)

An event-recording automaton $D = (\Sigma, L, l_0, \delta, L^f)$ is *deterministic* if

- $\delta(l, a, g)$ is a singleton set when it is defined
- if both $\delta(l, a, g_1)$ and $\delta(l, a, g_2)$ are both defined, then $[[g_1]] \cap [[g_2]] = \emptyset$

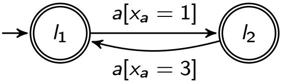
A guarded word $w_g = (a_1, g_1)(a_2, g_2) \cdots (a_n, g_n)$ is *accepted* by an ERA $D = (\Sigma, L, l_0, \delta, L^f)$ if

- $l_i = \delta(l_{i-1}, a_i, g_i)$ is defined for all $i \in \{1, 2, \ldots, n\}$
- $l_n \in L^f$

The *timed language* accepted by D, denoted by $\mathcal{L}(D)$, is the set of guarded words accepted by D.

Event-Recording Automata (cont.)

The following ERA $\mathcal{A}_1$ accepts the timed language $U_T = ((a, x_a = 1)(a, x_a = 3))^*$



$\mathcal{A}_1$

Outline

The TL* Algorithm

The TL* algorithm is a *timed* extension of the L* algorithm.

The TL* algorithm is a formal method to learn a minimal event-recording automaton (ERA) that accepts an unknown timed language U_T over an alphabet Σ

- We use U to denote the untimed language of U_T

The TL* Algorithm (cont.)

The TL* algorithm has to interact with a Minimal Adequate *Teacher*

- *untimed membership query* Q_m
  - Is an untimed word in the unknown untimed language U?
- *untimed candidate query* Q_c
  - Does a DFA accept the unknown untimed language U?
- *timed membership query* Q_m^T
  - Is a guarded word in the unkown timed language U_T?
- *timed candidate query* Q_c^T
  - Does an ERA accept the unknown timed language U_T?

The TL* Algorithm (cont.)

The TL* algorithm consists of two phases

- *Untimed Learning* Phase
  - The L* algorithm is used to learn a DFA M accepting the untimed language U
- *Timed Refinement* Phase
  - The DFA M is refined into an event-recording automaton (ERA) by adding time constraints

The TL* Algorithm (cont.)

input : Σ: alphabet, C_Σ: the set of event-recording clocks
output: a deterministic ERA accepting the unknown timed language U_T

Use L^* to learn a DFA M accepting $Untime(U_T)$;
Let (S, E, T) be the observation table during the L* learning process ;
Replace α by $(\alpha, true)$, s by $(s, true)$, and e by $(e, true)$ for each $\alpha \in \Sigma$, $s \in S$ and $e \in E$;
while *true* **do**
    if $Q_c^T(M) = 1$ **then** **return** M ;
    else
        Let $(a_1, g_1)(a_2, g_2) \cdots (a_n, g_n)$ be the counterexample given by Teacher ;
        foreach (a_i, g_i), $i \in \{1, 2, \ldots, n\}$ **do**
            if (a_i, g) *is a substring of* p *or* e *for some* $p \in S \cup (S \cdot \Sigma)$ *and* $e \in E$ *such that* $[[g_i]] \subset [[g]]$ **then**
                Let $G = \{\hat{g_1}, \hat{g_2}, \ldots, \hat{g_m}\}$ obtained by $[[g]] - [[g_i]]$;
                $\Sigma = \Sigma \setminus \{(a_i, g)\} \cup \{(a_i, g_i), (a_i, \hat{g_1}), (a_i, \hat{g_2}), \ldots, (a_i, \hat{g_m})\}$;
                Split p into $\{\hat{p_0}, \hat{p_1}, \hat{p_2}, \ldots, \hat{p_m}\}$ where (a_i, g_i) is a substring of $\hat{p_0}$ and $(a_i, \hat{g_j})$ is a substring of $\hat{p_j}$ for all $j \in \{1, 2, \ldots, m\}$;
                Split e into $\{\hat{e_0}, \hat{e_1}, \hat{e_2}, \ldots, \hat{e_m}\}$ where (a_i, g_i) is a substring of $\hat{e_0}$ and $(a_i, \hat{g_j})$ is a substring of $\hat{e_j}$ for all $j \in \{1, 2, \ldots, m\}$;
                Update T by $Q_{m_T}(\hat{p_j} \cdot \hat{e_j})$ for all $j \in \{0, 1, 2, \ldots, m\}$;

        while *there exists* $(s \cdot \alpha)$ *such that* $s \cdot \alpha \not\equiv s'$ *for all* $s' \in S$ **do**
            $S \longleftarrow S \cup \{s \cdot \alpha\}$;
            Update T by $Q_{m_T}((s \cdot \alpha) \cdot \beta)$ for all $\beta \in \Sigma$;

        $v \longleftarrow WS((a_1, g_1)(a_2, g_2) \cdots (a_n, g_n))$;
        if $|v| > 0$ **then**
            $E \longleftarrow E \cup \{v\}$;
            Update T by $Q_{m_T}(s \cdot v)$ and $Q_{m_T}(s \cdot \alpha \cdot v)$ for all $s \in S$ and $\alpha \in \Sigma$;

        Construct candidate M from (S, E, T) ;

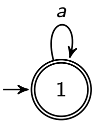
An Example

Suppose $U_T = ((a, x_a = 1)(a, x_a = 3))^*$ is the timed language to be learned.

Untimed Learning Phase

	λ
λ	1 (s_0)
a	1

(a) T_1



(b) M_1

	λ
λ	1 (s_0)
(a, $true$)	1

(c) T_2

$\mathcal{L}(M_1) = U = a^*$

An Example (cont.)

$Q_c^T(M_1) = 0$ with a negative counterexample $(a, x_a < 1)$

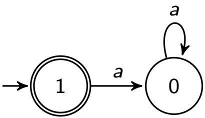
Timed Refinement 1

	λ
λ	1 (s_0)
$(a, x_a < 1)$	0
$(a, x_a \geq 1)$	0

(a) T_3

	λ
λ	1 (s_0)
$(a, x_a < 1)$	0 (s_1)
$(a, x_a \geq 1)$	0
$(a, x_a < 1)(a, x_a < 1)$	0
$(a, x_a < 1)(a, x_a \geq 1)$	0

(b) T_4



(c) M_2

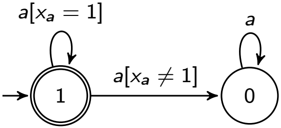
An Example (cont.)

$Q_c^T(M_2) = 0$ with a positive counterexample $(a, x_a = 1)$

Timed Refinement 2

	λ
λ	$1\ (s_0)$
$(a, x_a < 1)$	$0\ (s_1)$
$(a, x_a = 1)$	1
$(a, x_a > 1)$	0
$(a, x_a < 1)(a, x_a < 1)$	0
$(a, x_a < 1)(a, x_a = 1)$	0
$(a, x_a < 1)(a, x_a > 1)$	0

(a) T_5



(b) M_3

An Example (cont.)

$Q_c^T(M_3) = 0$ with a negative counterexample $(a, x_a = 1)(a, x_a = 1)$

A suffix $(a, x_a = 1)$ shows that λ and $(a, x_a = 1)$ should not be in the same class

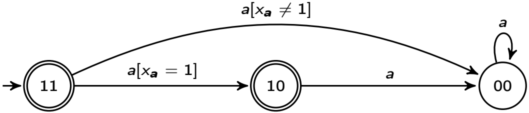
Timed Refinement 3

	λ	$(a, x_a = 1)$
λ	1	1 (s_0)
$(a, x_a < 1)$	0	0 (s_1)
$(a, x_a = 1)$	1	0
$(a, x_a > 1)$	0	0
$(a, x_a < 1)(a, x_a < 1)$	0	0
$(a, x_a < 1)(a, x_a = 1)$	0	0
$(a, x_a < 1)(a, x_a > 1)$	0	0

(a) T_6

	λ	$(a, x_a = 1)$
λ	1	1 (s_0)
$(a, x_a < 1)$	0	0 (s_1)
$(a, x_a = 1)$	1	0 (s_2)
$(a, x_a > 1)$	0	0
$(a, x_a < 1)(a, x_a < 1)$	0	0
$(a, x_a < 1)(a, x_a = 1)$	0	0
$(a, x_a < 1)(a, x_a > 1)$	0	0
$(a, x_a = 1)(a, x_a < 1)$	0	0
$(a, x_a = 1)(a, x_a = 1)$	0	0
$(a, x_a = 1)(a, x_a > 1)$	0	0

(b) T_7



(c) M_4

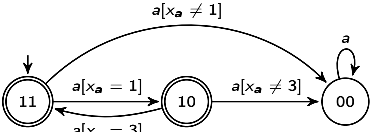
An Example (cont.)

$Q_c^T(M_4) = 0$ with a positive counterexample $(a, x_a = 1)(a, x_a = 3)$

Timed Refinement 4

	λ	$(a, x_a = 1)$
λ	1	1 (s_0)
$(a, x_a < 1)$	0	0 (s_1)
$(a, x_a = 1)$	1	0 (s_2)
$(a, 1 < x_a < 3)$	0	0
$(a, x_a = 3)$	0	0
$(a, x_a > 3)$	0	0
$(a, x_a < 1)(a, x_a < 1)$	0	0
$(a, x_a < 1)(a, x_a = 1)$	0	0
$(a, x_a < 1)(a, 1 < x_a < 3)$	0	0
$(a, x_a < 1)(a, x_a = 3)$	0	0
$(a, x_a < 1)(a, x_a > 3)$	0	0
$(a, x_a = 1)(a, x_a < 1)$	0	0
$(a, x_a = 1)(a, x_a = 1)$	0	0
$(a, x_a = 1)(a, 1 < x_a < 3)$	0	0
$(a, x_a = 1)(a, x_a = 3)$	1	1
$(a, x_a = 1)(a, x_a > 3)$	0	0

(a) T_8



(b) M_5

An Example (cont.)

$Q_c^T(M_5) = 1$, i.e., $\mathcal{L}(M_5) = U_T$

The learning process of TL* is finished

Analysis of TL*

Given a timed language U_T accepted by an ERA $\mathcal{A} = (\Sigma, L, l_0, \delta, L^f)$, the TL* algorithm needs to perform

- $O(|\Sigma| \cdot |G_\mathcal{A}| \cdot |L|^2 + |L| \log |\pi|)$ timed membership queries
  - π is the counterexample given by Teacher
  - $G_\mathcal{A}$ is the set of clock zones partitioned by the clock guards appearing in $\mathcal{A}$
- $O(|L| + |\Sigma| \cdot |G_\mathcal{A}|)$ timed candidate queries

Grinchtein's TL^*_{sg} needs $O(|\Sigma \times G_\Sigma| \cdot n^2 |\pi| \cdot |w| \binom{|\Sigma|+K}{|\Sigma|})$ timed membership queries

- n is the number of locations of the learned ERA
- w is the longest guarded word queried
- K is the largest constant appearing in the clock guards

Analysis of TL* (cont.)

Theorem

The TL algorithm is correct.*

Theorem

The TL algorithm terminates.*

Theorem

Assume the observation table (S, E, T) is closed and consistent and $M = (\Sigma, L, l^0, \delta, L^f)$ is the ERA constructed from the observation table (S, E, T). If $M' = (\Sigma, L', l^{0'}, \delta', L^{f'})$ is any other ERA consistent with T, then M' has at least $|L|$ locations.

Outline

Conclusion and Future Work

Conclusion

- We proposed an efficient polynomial time algorithm, TL$*$, for learning event-recording automata (ERA).
- The TL* algorithm has been implemented in the PAT model checker.

Future Work

- To extend the TL* algorithm to learn other subclasses of timed automata
- To automate assume-guarantee reasoning (AGR) for timed systems, we plan to use TL* to automatically generate timed assumptions needed for AGR.